1 Appendix

To solve the game, we first ask whether there are pooling or separating equilibria, where both types or one type, accepts the mediator's proposal. This establishes constraints on what the mediator can propose, and allows us to deduce the leader and enemy's sequentially rational best responses. From this we can determine the audience's beliefs and best response, since by Bayes' Rule, its beliefs must be consistent with all other players' strategies. Finally, we check that there are no profitable deviations on or off the equilibrium path to establish the equilibrium.

To assist the analysis, we assume that the mediator does not make an offer that obtains peace with probability zero. We prove that this assumption holds in equilibrium by showing that the mediator obtains peace with positive probability everywhere in the parameter space.

Assumption 1. *For any initial offer, m, if the probability of war is one, $P(war|m) = 1$, then the mediator does not propose m.*

1.1 Pooling on the high offer

Proposition 1 (Region I: High Offer). *When $p \leq p_H^*$, the mediator proposes $m^* = \sigma_H$, and both types of enemy accept, where $p_H^* = \frac{2c}{\tau_H - \tau_L}$. If the enemy accepts, the leader accepts m^* with beliefs $\lambda_1 = p$. If the enemy rejects, the leader's beliefs are $\lambda_2 \geq \frac{2c - \tau_H}{\tau_H - \tau_L}$, and the leader exits to war. The audience does not sanction the leader, $s^* = 0$, with beliefs $\alpha_L^m + \alpha_H^m = 1$ and $\alpha_L^r + \alpha_H^r = 0$. The probability of war is zero.*

Proof of Proposition 1. To see that there a pooling equilibrium in which both types accept, we can deduce a few things. First, for both types to accept, the mediator must propose at least the high offer, $m \geq \sigma_H$. Further, the leader must not raise, otherwise both types will reject m in favor of $m + \delta$, which means the leader must exit. Since the leader exits, a settlement is reached only through the mediator, and therefore consistency requires that the audience's beliefs are the mediator proposed the settlement, $\alpha_L^m + \alpha_H^m = 1$, and the leader did not raise, $\alpha_L^r + \alpha_H^r = 0$. Therefore, the audience does not sanction, $s^* = 0$.

What conditions are required to maintain these strategies? If the enemy rejects m, then rejection is off the equilibrium path. The leader knows that both types will accept any raised offer, since the mediator's offer is already high. Therefore, this can form an equilibrium only if there exists off-path beliefs, λ_2, such that the leader prefers to exit rather than secure a raised settlement. To refine the leader's off-path beliefs, we require that the equilibrium satisfy condition D1, which requires that the leader assign positive weight to the chance that the enemy is a high type, $\lambda_2 \neq 1$, and the mediator make the high offer, $m^* = \sigma_H$.[1] The leader will exit, rather than raise, if war provides a better payoff than the mediated settlement:

$$\begin{aligned} \lambda_2(-\tau_L - c) + (1 - \lambda_2)(-\tau_H - c) &\geq -(\tau_H - c) \\ \lambda_2(\tau_H - \tau_L) - c &\geq c - \tau_H \\ \lambda_2 &\geq \frac{2c - \tau_H}{\tau_H - \tau_L} \equiv \overline{\lambda_2}. \end{aligned} \tag{1}$$

The leader has a credible threat to exit as long as there is sufficient probability she faces a low type.

If the enemy accepts m^*, then since both types accept, the leader's beliefs are given

[1]We opt for the fewest restrictions on off-path beliefs. D1 requires that beliefs be supported on any type who stands to gain from deviation (Cho and Kreps, 1987). The low type never stands to gain from deviation, since knowing that the leader plans to exit, accepting the mediator's offer strictly dominates the low type's war payoff from rejecting it. Therefore, $\lambda_2 \neq 1$. Further, $m^* = \sigma_H$ because otherwise rejecting $m' > \sigma_H$ would be strictly dominated for the high type as well, and the leader could not assign positive weight to either type. Alternatively, universal divinity would result in the same high offer, $m^* = \sigma_H$, but would be more restrictive in needing more weight to be placed on the high type, $\lambda_2 < \frac{1}{2}$. The intuitive criterion would be even more restrictive in requiring that zero weight be put on the low type.

by her prior, $\lambda_1 = p$. Given this, the leader will accept m^* if:

$$-(\tau_H - c) \geq -p\tau_L - (1-p)\tau_H - c$$
$$p \leq \frac{2c}{\tau_H - \tau_L} \equiv p_H^* \tag{2}$$

The leader accepts the high offer, m^*, as long as there is sufficient probability she faces a high type. Since $\overline{\lambda_2} < p_H^*$, the leader's off-path beliefs are reasonable given her priors, and the above strategies can be supported. Since m^* guarantees peace, the mediator makes this proposal whenever $p \leq p_H^*$.

□

Lemma 1 (No Separating Equilibrium)**.** *There exists no separating equilibrium in which the low type accepts and the high type rejects the mediator's proposal.*

Proof of Lemma 1. To see that there is no separating equilibrium, suppose that the low type accepts an offer m and the high type rejects it. Then the leader believes that an enemy who accepts must be a low type, $\lambda_1 = 1$, and that an enemy who rejects must be a high type, $\lambda_2 = 0$. There are two possibilities: either the leader raises the offer, or exits to war. If the leader raises, then the low type will have a profitable deviation to reject m; thus, the leader must exit. However, if the leader exits, then settlement occurs only through the mediator, and by consistency, the audience does not sanction the leader, $s = 0$. To see that this is not an equilibrium, observe that the leader will raise as long as there exists some δ such that raising is preferred to exiting:

$$U_L(Exit|\lambda_2) \leq U_L(Raise|\lambda_2)$$
$$-\tau_H - c \leq -m - \delta$$
$$\delta \leq \tau_H + c - m.$$

For all $m < \tau_H + c$, since the leader believes the enemy is a high type, and knows the audience will not sanction, there exists some $\delta > 0$ such that the leader deviates to raise. The only way that the leader exits is if the mediator offers $m \geq \tau_H + c$, but then the high type profitably deviates to accept m, since $\tau_H + c > \tau_H - c$. □

1.2 Semi-separating equilibrium

The following lemmas specifies the best responses for each actor, before characterizing the semi-separating equilibria.

Lemma 2 (Enemy Response to m)**.** *In any semi-separating equilibrium, the low type must mix between accepting and rejecting m, while the high type rejects m.*

Proof of Lemma 2. To form a semi-separating equilibrium, it must be that the low type mixes between accepting and rejecting the mediator's offer, m, while the high type always rejects m. The reverse – for the high type to mix, and the low type to reject m – would not make sense.

To see this, let r represent the probability that the leader raises, and $1 - r$ the probability the leader exits. For the high type to mix, he must be indifferent between accepting and rejecting m, $U_{\tau_H}(accept\ m) = m = r(m+\delta)+(1-r)(\sigma_H) = U_{\tau_H}(reject\ m)$. But if that is true, then the low type will deviate to accept m, since for any r, m, and δ, the low type's payoff for rejecting m is strictly lower than the high type's, $U_{\tau_L}(reject\ m) = r(m + \delta) + (1 - r)(\sigma_L) < U_{\tau_H}(reject\ m)$, and therefore $m > U_{\tau_L}(reject\ m)$.

Further, it would not make sense for the low and high type to semi-separate in response to the leader's raise. That would require that the raised offer be equivalent to the low type's reservation value for war, $m + \delta = \sigma_L$, to make the low type indifferent. The probability of peace would be less than p, since the low type is mixing. But then, the mediator could make an offer in between the low type and leader's reservation values, $m \in (\sigma_L, \tau_L + c]$, that the low type would strictly prefer and the leader would be willing to accept. The mediator would strictly prefer this outcome in securing peace with probability p. Thus, semi-separation must occur about m.

Therefore, let q_L represent the probability that the low type accepts m, and $1 - q_L$ represent the probability the low type rejects m. □

Lemma 3 (Low Type)**.** *For the low type to mix between accepting and rejecting m, the leader must raise with probability $\overline{r} = \frac{m-\sigma_L}{m+\delta-\sigma_L}$.*

Proof of Lemma 3. For the low type to mix, the low type must be indifferent between accepting and rejecting m, $U_{\tau_L}(accept\ m|\cdot) = U_{\tau_L}(reject\ m|r,\delta)$. This section proves that for the low type to mix: 1. the leader must accept m following the enemy's acceptance of m, 2. the leader must raise with probability $\overline{r} = \frac{m-(\tau_L-c)}{m+\delta-(\tau_L-c)}$.

1. To see that the leader must accept m, consider the following proof by contradiction. Suppose that the leader rejects m. Then the low type knows that by accepting m, he receives his war payoff, σ_L. To keep the low type indifferent, the leader must not raise: if the leader raises with any positive probability r, then the low type would not be indifferent since a settlement $m+\delta$ with any positive probability is strictly preferred to war with certainty, $r(m+\delta)+(1-r)\sigma_L > \sigma_L$. But then war occurs with probability one, since when the enemy rejects m the leader also rejects, and when the enemy accepts m, the leader exits to war. By Assumption 1, this is not an equilibrium. Therefore, the leader must accept m.

2. To see that the leader must raise, consider the following. For the low type to mix, the low type must be indifferent between accepting and rejecting m. By the argument above, the low type will receive a utility of m if he accepts. Given this, the low type's indifference condition is:

$$\begin{aligned} U_{\tau_L}(accept\ m) &= U_{\tau_L}(reject\ m|r,\delta) \\ m &= r(m+\delta)+(1-r)(\tau_L-c) \end{aligned} \tag{1}$$

There are two ways to satisfy this indifference condition: either a) the leader never raises, $r=0$, and $m=\tau_L-c$; or b) the leader raises with positive probability that keeps the low type indifferent,

$$r = \frac{m-(\tau_L-c)}{m+\delta-(\tau_L-c)} \equiv \overline{r}. \tag{2}$$

To see that a) is not an equilibrium note that If the leader does not raise, $r = 0$, settlement is reached only through the mediator, and the audience will not sanction the leader. But then the leader can profitably deviate to raise, because:

$$
\begin{aligned}
U_L(exit|\lambda_2) &\leq U_L(raise|\lambda_2) \\
\lambda_2(-\tau_L - c) + (1-\lambda_2)(-\tau_H - c) &\leq -m - \delta \\
\lambda_2(\tau_H - \tau_L) - \tau_H - c &\leq -\tau_L + c - \delta \\
\lambda_2(\tau_H - \tau_L) &\leq \tau_H - \tau_L + 2c - \delta \\
\delta &\leq (\tau_H - \tau_L)(1-\lambda_2) + 2c.
\end{aligned}
$$

there exists $\delta > 0$ such that the leader prefers to deviate. Since a) is not an equilibrium, it must be that b) the leader raises with probability $\overline{r}$.

□

Lemma 4 (Leader's response to acceptance). *When the enemy accepts m, the leader will accept if $m \leq \overline{\sigma_L}$ with beliefs $\lambda_1 = 1$, where $\overline{\sigma_L} = \tau_L + c - sa(\sigma)$.*

Proof of Lemma 4. Upon observing the enemy accept m, the leader's beliefs are that the enemy must be a low type, $\lambda_1 = 1$. The leader's best response is to accept m if the mediator's proposal no greater than the leader's maximum settlement against the low type:

$$m \leq \tau_L + c - sa(\sigma) \equiv \overline{\sigma_L}. \tag{3}$$

□

Lemma 5 (Leader's response to rejection). *When the enemy rejects, the leader's beliefs that the enemy is a low type are $\lambda_2 = \frac{p(1-q_L)}{1-pq_L}$. The leader will mix between raising with $\delta^* = \sigma_H - m$ and exiting, if the low type accepts m with probability $\overline{q_L} = \frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{p[\tau_H-\tau_L-2c+sa(\sigma_H)]}$. The leader's beliefs when the low type plays this strategy is $\lambda_2 = \frac{2c-sa(\sigma_H)}{\tau_H-\tau_L}$.*

Proof of Lemma 5. When the enemy rejects m, then the leader updates her beliefs that 2 is a low type, λ_2. Since the low type rejects m with probability $1 - q_L$, and the high type always rejects m, the leader's posterior belief that the enemy is a low type is:

$$\lambda_2 = \frac{p(1-q_L)}{p(1-q_L) + 1 - p} = \frac{p(1-q_L)}{1 - pq_L}. \tag{4}$$

If the leader raises, she must raise with $\delta^* = \sigma_H - m$. Why? The leader will not raise with anything higher, $\delta' > \delta^*$, because then the leader overpays for peace against both types. The leader will not raise with anything lower, $\delta' < \delta^*$, because then the leader overpays for peace against the low type: since m is acceptable to the low type, the leader can offer any $\delta > 0$ and secure peace against the low type for a lower price, thus any $\delta' < \delta^*$ cannot form an equilibrium (the leader can always deviate to ϵ lower).[2] Therefore, the only reason for the leader to raise is to change the outcome by securing peace against the high type with $\delta^* = \sigma_H - m$. By sequential rationality, both types will accept this raised offer.

We can now plug these components into the leader's indifference condition. If the leader exits to war, she fights either the low or high type, and if the leader raises, then she offers a total settlement σ_H and pays audience costs with probability s:

$$\begin{aligned}
U_L(exit|\lambda_2) &= U_L(raise|\lambda_2, m + \delta^* = \sigma_H) \\
\lambda_2(-\tau_L - c) + (1-\lambda_2)(-\tau_H - c) &= -\sigma_H - sa(\sigma_H) \\
\lambda_2(\tau_H - \tau_L) - \tau_H - c &= -\tau_H + c - sa(\sigma_H) \\
\lambda_2 &= \frac{2c - sa(\sigma_H)}{\tau_H - \tau_L} \equiv p_H^* - \frac{sa(\sigma_H)}{\tau_H - \tau_L}.
\end{aligned} \tag{5}$$

Given the leader's beliefs, λ_2, from (4), we can rearrange the leader's indifference condition

[2] By definition, it would not make sense for the leader to "back down" by offering zero concessions, thus, we do not allow $\delta = 0$.

as follows:

$$
\begin{aligned}
\frac{p(1-q_L)}{1-pq_L} &= \frac{2c - sa(\sigma_H)}{\tau_H - \tau_L} \\
p(1-q_L)(\tau_H-\tau_L) &= (2c - sa(\sigma_H))(1-pq_L) \\
p(\tau_H-\tau_L) - pq_L(\tau_H-\tau_L) &= 2c - sa(\sigma_H) - pq_L(2c - sa(\sigma_H)) \\
p(\tau_H-\tau_L) - 2c + sa(\sigma_H) &= pq_L[\tau_H - \tau_L - 2c + sa(\sigma_H)] \\
\overline{q_L} &= \frac{p(\tau_H-\tau_L) - 2c + sa(\sigma_H)}{p[\tau_H - \tau_L - 2c + sa(\sigma_H)]}. \qquad (6)
\end{aligned}
$$

This indicates that for the leader to be indifferent, the low type must accept m with probability $\overline{q_L} = \frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{p[\tau_H-\tau_L-2c+sa(\sigma_H)]}$.

□

Lemma 6 (Audience)**.** *The audience's best response is sanction if $m > \widehat{\sigma_L}$, to not sanction if $m < \widehat{\sigma_L}$, with beliefs $\alpha_H^m = 0$, and α_L^m, α_L^r, and α_H^r given by (7), (8), and (9). The audience is indifferent when $m = \widehat{\sigma_L}$, where $\widehat{\sigma_L} = \frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{1-p} + \tau_L - c$, with beliefs $\alpha_H^m = 0$, and $\alpha_L^m = \alpha_L^r + \alpha_H^r = \frac{1}{2}$.*

Proof of Lemma 6. Given Lemmas 3, 4, and 5 settlement occurs on the equilibrium path. The audience updates its beliefs that upon observing a settlement, and believes that under no condition has the high type accepted the mediator's offer, $\alpha_H^m = 0$. The audience believes that the low type accepted the mediator's offer with probability:

$$\alpha_L^m = \frac{pq_L}{pq_L + p(1-q_L)r + (1-p)r}, \qquad (7)$$

the low type accepted the leader's raised offer with probability

$$\alpha_L^r = \frac{p(1-q_L)r}{pq_L + p(1-q_L)r + (1-p)r}, \qquad (8)$$

and the high type accepted the leader's raised offer with probability

$$\alpha_H^r = \frac{(1-p)r}{pq_L + p(1-q_L)r + (1-p)r}. \qquad (9)$$

Given these beliefs, the audience's best response is to sanction if the following holds:

$$
\begin{aligned}
U_A(sanction|\cdot) &\geq U_A(not\ sanction|\cdot) \\
\alpha_L^r + \alpha_H^r &\geq \alpha_L^m + \alpha_H^m \\
p(1-q_L)r + (1-p)r &\geq pq_L \\
pr - q_L pr + r - pr &\geq pq_L \\
r - q_L pr &\geq pq_L \\
r &\geq \frac{pq_L}{1-pq_L}. \qquad (10)
\end{aligned}
$$

Since we know the probability that the low type accepts m, $\overline{q_L}$, and the probability the leader raises, $\overline{r}$, we can plug these values into (10) to determine the audience's best response.

Substitution of $\overline{q_L} = \frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{p[\tau_H-\tau_L-2c+sa(\sigma_H)]}$ gives

$$
\begin{aligned}
r &\geq \frac{\frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{\tau_H-\tau_L-2c+sa(\sigma_H)}}{1-\frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{\tau_H-\tau_L-2c+sa(\sigma_H)}} \\
&\geq \frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{\tau_H-\tau_L-2c+sa(\sigma_H)-p(\tau_H-\tau_L)+2c-sa(\sigma_H)} \\
&\geq \frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{(\tau_H-\tau_L)(1-p)}.
\end{aligned}
$$

Then, substitution of $\overline{r} = \frac{m-\tau_L+c}{\tau_H-\tau_L}$ gives

$$
\begin{aligned}
\frac{m-\tau_L+c}{\tau_H-\tau_L} &\geq \frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{(\tau_H-\tau_L)(1-p)} \\
m-\tau_L+c &\geq \frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{1-p} \\
m &\geq \frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{1-p} + \tau_L - c \equiv \widehat{\sigma_L}. \qquad (11)
\end{aligned}
$$

The audience's best response is sanction if $m > \widehat{\sigma_L}$, not to sanction if $m \leq \widehat{\sigma_L}$, and to be indifferent if $m = \widehat{\sigma_L}$, where $\widehat{\sigma_L} = \frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{1-p} + \tau_L - c$. □

Lemma 7 (Mediator)**.** *The mediator's best response is to offer* $m^* = \min\{\overline{\sigma_L}, \widehat{\sigma_L}\}$*, which means the mediator offers* $m^* = \overline{\sigma_L}$ *when* $p > p_L^*$*, and offers* $m^* = \widehat{\sigma_L}$ *when* $p < p_L^*$*, where*

$p_L^* = \frac{4c}{\tau_H - \tau_L + 2c}$. *The probability of war is* $P(war) = \frac{(1-p)(\tau_H - m - c)}{\tau_H - \tau_L - 2c + sa(\sigma_H)}$.

Proof of Lemma 7. Given these best responses, the mediator makes a proposal that minimizes the probability of war. War occurs in two ways. Either the enemy is a low type who rejected m, and the leader did not raise, or the enemy is a high type who rejected m, and the leader did not raise. Therefore, the probability of war is:

$$P(war) = p(1 - q_L)(1 - r) + (1 - p)(1 - r), \tag{12}$$

which reduces to $P(war) = (1 - r)(1 - pq_L)$. Substitution of $\overline{r}$ and $\overline{q_L}$ gives:

$$\begin{aligned} P(war) &= \left(1 - \frac{m - \tau_L + c}{\tau_H - \tau_L}\right)\left(1 - \frac{p(\tau_H - \tau_L) - 2c + sa(\sigma_H)}{\tau_H - \tau_L - 2c + sa(\sigma_H)}\right) \\ &= \left(\frac{\tau_H - m - c}{\tau_H - \tau_L}\right)\left(\frac{(\tau_H - \tau_L)(1 - p)}{\tau_H - \tau_L - 2c + sa(\sigma_H)}\right) \\ &= \frac{(1 - p)(\tau_H - m - c)}{\tau_H - \tau_L - 2c + sa(\sigma_H)}. \end{aligned} \tag{13}$$

The probability of war is decreasing in m and s. If $s = 0$, then the probability of war is only decreasing in m, and the mediator proposes the most that the leader will tolerate, $m^* = \overline{\sigma_L}$. In order for $s = 0$, by Lemma 6, the audience will not sanction if $m^* \leq \widehat{\sigma_L}$, which is true if:

$$\begin{aligned} \overline{\sigma_L} &\leq \widehat{\sigma_L} \\ \tau_L + c &\leq \frac{p(\tau_H - \tau_L) - 2c}{1 - p} + \tau_L - c \\ (2c)(1 - p) &\leq p(\tau_H - \tau_L) - 2c \\ 4c &\leq p(\tau_H - \tau_L + 2c) \\ p &\geq \frac{4c}{\tau_H - \tau_L + 2c} \equiv p_L^*, \end{aligned} \tag{14}$$

where $p_L^* > p_H^*$ since $2c < \tau_H - \tau_L$.[3] Therefore, when $p \geq p_L^*$, the mediator offers $m^* = \overline{\sigma_L}$

[3]

$$p_L^* > p_H^*$$

and the audience does not sanction, $s^* = 0$. Let us call this Region II. Proposition 2 specifies this equilibrium.

When $p \in (p_H^*, p_L^*)$, the relationship between $\overline{\sigma_L}$ and $\widehat{\sigma_L}$ depends on s. In other words, there exists an s such that these values are equal. This means that mediator can either offer: 1) $m = \overline{\sigma_L} > \widehat{\sigma_L}$, which gets the leader sanctioned, $s = 1$, 2) $m = \widehat{\sigma_L} < \overline{\sigma_L}$, which keeps the audience indifferent, or 3) $m = \widehat{\sigma_L} = \overline{\sigma_L}$, which also keeps the audience indifferent but is the highest offer the leader will tolerate. We examine each in Lemmas 8, 9, and 10. Let us call this Region III. Proposition 3 specifies this equilibrium.

□

Proposition 2 (Region II: No Sanction). *When $p \geq p_L^*$, the mediator offers $m^* = \tau_L + c$, where $p_L^* = \frac{4c}{\tau_H - \tau_L + 2c}$. The low type accepts with probability $q_L^* = \frac{p(\tau_H - \tau_L) - 2c}{p(\tau_H - \tau_L - 2c)}$, and the high type rejects. If the enemy accepts, the leader accepts m^* with beliefs $\lambda_1 = 1$. If the enemy rejects, the leader raises with probability $r^* = \frac{2c}{\tau_H - \tau_L}$ to offer $\delta^* = \tau_H - \tau_L - 2c$ with beliefs $\lambda_2 = \frac{2c}{\tau_H - \tau_L}$. Both types accept the raised offer. The audience does not sanction, $s^* = 0$, with beliefs $\alpha_H^m = 0$, and $\alpha_L^m > \alpha_L^r + \alpha_H^r$. The probability of war is $1 - p$.*

Proof of Proposition 2. When $p > p_L^*$, $\overline{\sigma_L} < \widehat{\sigma_L}$, which implies $\alpha_L^m > \alpha_L^r + \alpha_H^r$. Therefore, by Lemma 6, the audience does not sanction, $s^* = 0$, and the mediator offers $m^* = \overline{\sigma_L} = \tau_L + c$. By Lemma 5, the low type accepts m^* with probability $q_L^* = \frac{p(\tau_H - \tau_L) - 2c}{p(\tau_H - \tau_L - 2c)}$, which maintains the leader's indifference, while the high type rejects m. By Lemma 4, the leader updates her beliefs, $\lambda_1 = 1$, and accepts m, since this meets her reservation value against the low type, $m = \overline{\sigma_L}$. By Lemmas 3 and 5, when the enemy rejects m, the leader updates her beliefs, $\lambda_2 = \frac{2c}{\tau_H - \tau_L}$, and raises with probability $r^* = \frac{2c}{\tau_H - \tau_L}$, and offers

$$\frac{4c}{\tau_H - \tau_L + 2c} > \frac{2c}{\tau_H - \tau_L}$$
$$\frac{2}{\tau_H - \tau_L + 2c} > \frac{1}{\tau_H - \tau_L}$$
$$2(\tau_H - \tau_L) > \tau_H - \tau_L + 2c$$
$$\tau_H - \tau_L > 2c$$

$\delta^* = \tau_H - \tau_L - 2c$, which maintains the low type's indifference. Both types accept the raised offer, since $m^* + \delta^* = \sigma_H$. By Lemma 7, the probability of war is

$$P(war) = \frac{(1-p)(\tau_H - m^* - c)}{\tau_H - \tau_L - 2c + s^* a(\sigma_H)} \tag{15}$$
$$= \frac{(1-p)(\tau_H - \tau_L - 2c)}{\tau_H - \tau_L - 2c} \tag{16}$$
$$= 1 - p. \tag{17}$$

□

Lemma 8 (Region II: Audience Sanctions, $m = \overline{\sigma_L} > \widehat{\sigma_L}$)**.** *When $p < p_L^*$, and the audience is weak, $a(\sigma_H) < 2c$, then the mediator can offer $m = \tau_L + c - a(\sigma)$. The low type accepts with probability $\overline{q_L} = \frac{p(\tau_H - \tau_L) - 2c + a(\sigma_H)}{p[\tau_H - \tau_L - 2c + a(\sigma_H)]}$, the high type rejects. If the enemy accepts, the leader accepts with beliefs $\lambda_1 = 1$. If the enemy rejects, the leader believes it is a low type with probability $\lambda_2 = \frac{2c - a(\sigma_H)}{\tau_H - \tau_L}$, and raises with probability $\overline{r} = \frac{2c - a(\sigma)}{\tau_H - \tau_L}$ and concessions $\delta = \tau_H - \tau_L - 2c + a(\sigma)$. Both types accept the raised offer. The audience sanctions the leader, $s = 1$. The probability of war is $\frac{(1-p)[\tau_H - \tau_L - 2c + a(\sigma)]}{\tau_H - \tau_L - 2c + a(\sigma_H)}$ which is less than $1 - p$. This forms an equilibrium as long as the mediator does not prefer another offer.*

Proof of Lemma 8. When $p < p_L^*$, the mediator can offer $m = \overline{\sigma_L} = \tau_L + c - a(\sigma)$, and by Lemma 6, the audience will sanction the leader, $s = 1$. If $s = 1$, then by Lemma 5, the low type must accept with probability $\overline{q_L} = \frac{p(\tau_H - \tau_L) - 2c + a(\sigma_H)}{p[\tau_H - \tau_L - 2c + a(\sigma_H)]}$ for the leader to mix. The low type is willing to accept m as long as $\tau_L + c - a(\sigma) > \tau_L - c$, which is true if $2c > a(\sigma)$. The high type rejects m.

If the enemy accepts m, then by Lemma 4, the leader believes the enemy is a low type with probability $\lambda_1 = 1$, and accepts since m is the most she will tolerate against the low type. If the enemy rejects, then by Lemma 5, the leader's beliefs are $\lambda_2 = \frac{2c - a(\sigma_H)}{\tau_H - \tau_L}$, where $\lambda_2 \in [0, 1]$ if $2c > a(\sigma_H)$. We will refer to this as the "weak audience" requirement.[4]

[4]Note that if $2c > a(\sigma_H)$, then $2c > a(\sigma)$, since $a(\sigma_H) > a(\sigma)$. Therefore, both the low type and leader's strategies are maintained if the weak audience requirement is satisfied.

By Lemma 3, the leader must raise with probability $\overline{r} = \frac{2c-a(\sigma)}{\tau_H-\tau_L}$ and concessions $\delta = \tau_H - \tau_L - 2c + a(\sigma)$ to keep the low type indifferent, where $\delta > 0$ since $2c < \tau_H - \tau_L$. Both types accept the raised offer.

These strategies accord with the best responses required for a semi-separating equilibrium. By Lemma 7, the probability of war is:

$$P(war) = \frac{(1-p)[\tau_H - \tau_L - 2c + a(\sigma)]}{\tau_H - \tau_L - 2c + a(\sigma_H)},$$

which is less than $1-p$ since $a(\sigma) < a(\sigma_H)$. This forms an equilibrium as long as the mediator does not prefer another offer. $\square$

Lemma 9 (Region III: Audience Indifferent, $m = \widehat{\sigma_L} < \overline{\sigma_L}$)**.** *When $p < \min\{p_L^*, \frac{1}{2}\}$, if the audience is sufficiently strong, $a(\sigma_H) > 2c$, and sanctioning for accepting the mediator's offer is not too high, $a(\sigma) \leq 2c - \frac{p(\tau_H-\tau_L)}{1-p}$, then the mediator can offer $m = \sigma_L + \frac{p(\tau_H-\tau_L)}{1-p}$. The low type accepts, $\overline{q_L} = 1$, the high type rejects. If the enemy accepts, then the leader accepts the mediator's offer with beliefs $\lambda_1 = 1$. If the enemy rejects, the leader raises with probability $\overline{r} = \frac{p}{1-p}$ and concessions $\delta = \frac{(1-2p)(\tau_H-\tau_L)}{1-p}$ with beliefs $\lambda_2 = 0$. The audience sanctions with probability $s_H^* = \frac{2c}{a(\sigma_H)}$ and beliefs $\alpha_L^m = \alpha_H^r = \frac{1}{2}$, $\alpha_L^r = 0$, and $\alpha_H^m = 0$. The probability of war is $1-2p$. This forms an equilibrium as long as the mediator does not prefer another offer.*

Proof of Lemma 9. Alternatively, when $p < p_L^*$, the mediator can offer $m = \widehat{\sigma_L} = \frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{1-p} + \tau_L - c$, which makes the audience indifferent. However, notice that m is a function of s.

Therefore, plugging m into the $P(war)$ will yield an expression for the $P(war)$ that only depends on s:

$$\begin{aligned} P(war|\widehat{\sigma_L}) &= \frac{(1-p)\left(\tau_H - c - \left(\frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{1-p} + \tau_L - c\right)\right)}{\tau_H - \tau_L - 2c + sa(\sigma_H)} \\ &= \frac{(1-p)(\tau_H-\tau_L) - p(\tau_H-\tau_L) + 2c - sa(\sigma_H)}{\tau_H - \tau_L - 2c + sa(\sigma_H)} \\ &= \frac{(1-2p)(\tau_H-\tau_L) + 2c - sa(\sigma_H)}{\tau_H - \tau_L - 2c + sa(\sigma_H)}. \end{aligned} \tag{18}$$

This is decreasing in s, which can be seen by sketching a similar function, $y = \frac{1-x}{2+x}$, or by taking the derivative of the $P(war|\widehat{\sigma_L})$ with respect to s.[5] Therefore, the mediator's best option is to offer the value of m that corresponds to the maximum value of s.

The maximum value of s is given by constraints in the best responses of the other actors, which also depend on s. The low type accepts m with probability $\overline{q_L} = \frac{p(\tau_H - \tau_L) - 2c + sa(\sigma_H)}{p[\tau_H - \tau_L - 2c + sa(\sigma_H)]}$, which requires that $p(\tau_H - \tau_L) - 2c + sa(\sigma_H) > 0$. The leader accepts if $m^* < \overline{\sigma_L} = \tau_L + c - sa(\sigma)$. If the enemy rejects, the leader raises with probability $\overline{r} = \frac{p(\tau_H - \tau_L) - 2c + sa(\sigma_H)}{(1-p)(\tau_H - \tau_L)}$,[6] with additional concessions $\delta^* = \frac{(1-2p)(\tau_H - \tau_L) + 2c - sa(\sigma_H)}{1-p}$,[7] and beliefs $\lambda_2 = \frac{2c - sa(\sigma_H)}{\tau_H - \tau_L}$.

Given these strategies, the constraints on the audience's probability of sanctioning

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[5]Using the quotient rule, where $\frac{\delta[(1-2p)(\tau_H - \tau_L) + 2c - sa(\sigma_H)]}{\delta s} = -a(\sigma_H)$ and $\frac{\delta[\tau_H - \tau_L - 2c + sa(\sigma_H)]}{\delta s} = a(\sigma_H)$, we obtain:

$$\begin{aligned}\frac{\delta P(war|\widehat{\sigma_L})}{\delta s} &= \frac{-a(\sigma_H)[\tau_H - \tau_L - 2c + sa(\sigma_H)] - a(\sigma_H)[(1-2p)(\tau_H - \tau_L) + 2c - sa(\sigma_H)]}{[\tau_H - \tau_L - 2c + sa(\sigma_H)]^2} \\ &= \frac{-a(\sigma_H)[2(1-p)(\tau_H - \tau_L)]}{[\tau_H - \tau_L - 2c + sa(\sigma_H)]^2} < 0.\end{aligned}$$

Since the denominator is positive, $\tau_H - \tau_L - 2c + sa(\sigma_H) > 0$, we know that the derivative of $P(war|\widehat{\sigma_L})$ with respect to s is negative. Therefore, $P(war|\widehat{\sigma_L})$ is minimized by the maximum value of s.

[6]By Lemma 3, the leader must raise with the following probability to maintain the low type's indifference:

$$\begin{aligned}\overline{r} &= \frac{m^*_{\widehat{\sigma_L}} - \tau_L + c}{\tau_H - \tau_L} \\ &= \frac{\frac{p(\tau_H - \tau_L) - 2c + sa(\sigma_H)}{1-p} + \tau_L - c - \tau_L + c}{\tau_H - \tau_L} \\ &= \frac{p(\tau_H - \tau_L) - 2c + sa(\sigma_H)}{(1-p)(\tau_H - \tau_L)}.\end{aligned}$$

[7]By Lemma 5, the leader raises with additional concessions given by:

$$\delta^*_{\widehat{\sigma_L}} = \sigma_H - m^*_{\widehat{\sigma_L}}$$

can be summarized as follows:

$$\lambda_2 \in [0,1] \longleftrightarrow s \in \left[\frac{2c-(\tau_H-\tau_L)}{a(\sigma_H)}, \frac{2c}{a(\sigma_H)}\right], \tag{19}$$

and

$$\overline{r} \in [0,1] \longleftrightarrow s \in \left[\frac{2c-p(\tau_H-\tau_L)}{a(\sigma_H)}, \frac{2c+(1-2p)(\tau_H-\tau_L)}{a(\sigma_H)}\right], \tag{20}$$

where one can verify that these constraints also satisfy $m^* > \sigma_L$, $\overline{q_L} \in [0,1]$, and $\delta > 0$.

These constraints are ordered thus creating two possibilities.[8]

1. When a high type is more likely, $p < \frac{1}{2}$, the audience may sanction with any probability in the following range to maintain this equilibrium. Let us denote the set of probabilities s_H.

$$s_H = \left\{s : \frac{2c-p(\tau_H-\tau_L)}{a(\sigma_H)} < s < \frac{2c}{a(\sigma_H)}\right\}.$$

2. When a low type is more likely, $p > \frac{1}{2}$, the audience may sanction with any prob-

$$\begin{aligned}
&= \tau_H - c - \left(\frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{1-p} + \tau_L - c\right)\\
&= \tau_H - \tau_L - \frac{p(\tau_H-\tau_L)-2c+sa(\sigma_H)}{1-p}\\
&= \frac{(1-2p)(\tau_H-\tau_L)+2c-sa(\sigma_H)}{1-p}.
\end{aligned}$$

[8]We know that $\frac{2c-(\tau_H-\tau_L)}{a(\sigma_H)} < \frac{2c-p(\tau_H-\tau_L)}{a(\sigma_H)} < \frac{2c}{a(\sigma_H)}$, since $p \in [0,1]$. Further, when $p < \frac{1}{2}$, we know that $1-2p > 0$, and therefore $\frac{2c}{a(\sigma_H)} < \frac{2c+(1-2p)(\tau_H-\tau_L)}{a(\sigma_H)}$. This gives the ordering for possibility 1: $\frac{2c-(\tau_H-\tau_L)}{a(\sigma_H)} < \frac{2c-p(\tau_H-\tau_L)}{a(\sigma_H)} < \frac{2c}{a(\sigma_H)} < \frac{2c+(1-2p)(\tau_H-\tau_L)}{a(\sigma_H)}$. When $p > \frac{1}{2}$, $1-2p < 0$, and therefore, $\frac{2c+(1-2p)(\tau_H-\tau_L)}{a(\sigma_H)} < \frac{2c}{a(\sigma_H)}$. Further, it can be shown that $\frac{2c-p(\tau_H-\tau_L)}{a(\sigma_H)} < \frac{2c+(1-2p)(\tau_H-\tau_L)}{a(\sigma_H)}$ for all $p < 1$, and when $p = 1$, $\frac{2c-p(\tau_H-\tau_L)}{a(\sigma_H)} = \frac{2c+(1-2p)(\tau_H-\tau_L)}{a(\sigma_H)}$. This gives the ordering for possibility 2: $\frac{2c-(\tau_H-\tau_L)}{a(\sigma_H)} < \frac{2c-p(\tau_H-\tau_L)}{a(\sigma_H)} < \frac{2c+(1-2p)(\tau_H-\tau_L)}{a(\sigma_H)} < \frac{2c}{a(\sigma_H)}$.

ability in the following range to maintain this equilibrium. Let us denote this set s_L.

$$s_L = \Big\{ s : \frac{2c - p(\tau_H - \tau_L)}{a(\sigma_H)} < s < \frac{2c + (1-2p)(\tau_H - \tau_L)}{a(\sigma_H)} \Big\}.$$

Since the mediator chooses the offer that induces the highest probability of sanctioning, $s_H^* = \frac{2c}{a(\sigma_H)}$, and $s_L^* = \frac{2c+(1-2p)(\tau_H-\tau_L)}{a(\sigma_H)}$.

One can quickly show that s_L^* does not form an equilibrium. Substitution indicates that the mediator's offer is $m^* = \frac{p(\tau_H-\tau_L)+(1-2p)(\tau_H-\tau_L)}{1-p} + \tau_L - c = \tau_H - c$. The leader accepts m^* if

$$\tau_H - c \leq \tau_L + c - sa(\sigma)$$
$$\tau_H - \tau_L - 2c \leq -sa(\sigma).$$

Since $2c < \tau_H - \tau_L$, the left side of this equation is positive, while the right side is negative. Therefore, the leader rejects this offer, and s_L^* is not in equilibrium.

s_H^* forms an equilibrium: When $p < \frac{1}{2}$, the audience sanctions with probability $s_H^* = \frac{2c}{a(\sigma_H)}$, and the mediator offers $m = \frac{p(\tau_H-\tau_L)}{1-p} + \tau_L - c = \sigma_L + \frac{p(\tau_H-\tau_L)}{1-p}$. For $s_H^* \in (0,1)$, the audience must be sufficiently strong, $a(\sigma_H) > 2c$. We will refer to this as the "strong audience" requirement.

By Lemma 5, the low type must accept m with probability $\overline{q_L} = 1$ to maintain the leader's indifference. The leader's beliefs are $\lambda_1 = 1$ and $\lambda_2 = 0$. The leader accepts the mediator's offer as long as:

$$m \leq \tau_L + c - a(\sigma)s_H^*$$
$$\sigma_L + \frac{p(\tau_H - \tau_L)}{1-p} \leq \tau_L + c - a(\sigma)\left[\frac{2c}{a(\sigma_H)}\right].$$

Since this must hold for $a(\sigma_H) > 2c$, the right side of the inequality is strictly greater than $\tau_L + c - a(\sigma)$. Therefore, a sufficient condition for the above to hold is:

$$\sigma_L + \frac{p(\tau_H - \tau_L)}{1-p} \leq \tau_L + c - a(\sigma)$$

$$a(\sigma) \leq 2c - \frac{p(\tau_H - \tau_L)}{1-p}.$$

The sanction for accepting the mediator's punishment must be sufficiently low.

By Lemma 5, the leader must raise with probability $\overline{r} = \frac{p(\tau_H-\tau_L)}{(1-p)(\tau_H-\tau_L)} = \frac{p}{1-p}$, with additional concessions $\delta = \frac{(1-2p)(\tau_H-\tau_L)}{1-p}$. Given these strategies, the audience's beliefs are $\alpha_L^m = \frac{p}{p+(1-p)r} = \frac{p}{p+(1-p)\left[\frac{p(\tau_H-\tau_L)}{(1-p)(\tau_H-\tau_L)}\right]} = \frac{1}{2}$, $\alpha_L^r = 0$, $\alpha_H^m = 0$, $\alpha_H^r = \frac{1}{2}$, which maintains its indifference. By (18), the probability of war is $P(war) = \frac{(1-2p)(\tau_H-\tau_L)}{\tau_H-\tau_L} = 1-2p$. This is an equilibrium possibility for p where $p < \min\{p_L^*, \frac{1}{2}\}$ as long as the mediator does not prefer another offer. $\square$

Lemma 10 (Region III: Audience Indifferent, $m = \widehat{\sigma_L} = \overline{\sigma_L}$)**.** *When $p < p_L^*$, the mediator can offer $m = \tau_L + c - a(\sigma)s_G^*$. The low type accepts with probability $\overline{q_L} = \frac{p(\tau_H-\tau_L)-2c+a(\sigma_H)s_G^*}{p\left[\tau_H-\tau_L-2c+a(\sigma_H)s_G^*\right]}$, and the high type rejects. If the enemy accepts, then the leader accepts the mediator's offer with beliefs $\lambda_1 = 1$. If the enemy rejects, the leader raises with probability $\overline{r} = \frac{2c-a(\sigma)s_G^*}{\tau_H-\tau_L}$ and concessions $\delta = \tau_H - \tau_L - 2c + a(\sigma)s_G^*$ with beliefs $\lambda_2 = \frac{2c-a(\sigma_H)s_G^*}{\tau_H-\tau_L}$. The audience sanctions with probability $s_G^* = \frac{4c-p[\tau_H-\tau_L+2c]}{a(\sigma_H)+a(\sigma)(1-p)}$ and beliefs $\alpha_L^m = \frac{1}{2}$, $\alpha_L^r + \alpha_H^r = \frac{1}{2}$, and $\alpha_H^m = 0$. The probability of war is $1-p$. This forms an equilibrium as long as the mediator does not prefer another offer.*

Proof of Lemma 10. If $p < p_L^*$ and $p > \frac{1}{2}$, or $p < p_L^*$ and the weak or strong audience requirements are not met, then the best that the mediator can do is to minimize the probability of war given the highest value of $m = \widehat{\sigma_L}$ that the leader is willing to accept, $m = \widehat{\sigma_L} \leq \overline{\sigma_L}$. Note that this is different from Lemma 9: here the mediator minimizes the probability of war subject to the maximum the leader will accept, which permits some value $s \in (0,1)$ but not necessarily the highest permissible value of s. Therefore, since the probability of war is decreasing in s, this will yield a larger probability of war than the equilibrium in Lemma 9. While this is worse for the mediator than the options in Lemmas 8 and 9, this is the best that the mediator can do, since any offer of m for which $s = 0$ in this region would make war more likely.

To solve for this value of s, the leader accepts $m = \widehat{\sigma_L}$ if:

$$
\begin{aligned}
\widehat{\sigma_L} &\leq \overline{\sigma_L} \\
\frac{p(\tau_H - \tau_L) - 2c + sa(\sigma_H)}{1-p} + \tau_L - c &\leq \tau_L + c - sa(\sigma) \\
\frac{p(\tau_H - \tau_L) - 2c}{1-p} + \frac{sa(\sigma_H)}{1-p} + sa(\sigma) &\leq 2c \\
s\left[\frac{a(\sigma_H)}{1-p} + a(\sigma)\right] &\leq 2c - \frac{p(\tau_H - \tau_L) - 2c}{1-p} \\
&\leq \frac{2c(2-p) - p(\tau_H - \tau_L)}{1-p} \\
s\left[\frac{a(\sigma_H) + a(\sigma)(1-p)}{1-p}\right] &\leq \frac{4c - p[\tau_H - \tau_L + 2c]}{1-p} \\
s &\leq \frac{4c - p[\tau_H - \tau_L + 2c]}{a(\sigma_H) + a(\sigma)(1-p)} \equiv s_G^*,
\end{aligned}
$$

which makes sense, since as we move toward Region II, $p \to p_L^*$, the audience's strategy converges to its equilibrium strategy in Region II, $s_G^* \to 0$.

Note that substitution of s_G^* into $m = \widehat{\sigma_L}$, where by construction $\widehat{\sigma_L} = \overline{\sigma_L}$, gives:

$$
m = \tau_L + c - a(\sigma)\left[\frac{4c - p[\tau_H - \tau_L + 2c]}{a(\sigma_H) + a(\sigma)(1-p)}\right].
$$

Additional substitution of s_G^* into the semi-separating equilibrium's best responses in Lemmas 3 to 7 establishes this lemma.

By Lemma 5, the leader mixes her strategies if the low type accepts m with probability:

$$
\begin{aligned}
\overline{q_L} &= \frac{p(\tau_H - \tau_L) - 2c + sa(\sigma_H)}{p[\tau_H - \tau_L - 2c + sa(\sigma_H)]} \\
&= \frac{p(\tau_H - \tau_L) - 2c + a(\sigma_H)\left[\frac{4c - p[\tau_H - \tau_L + 2c]}{a(\sigma_H) + a(\sigma)(1-p)}\right]}{p\left[\tau_H - \tau_L - 2c + a(\sigma_H)\left[\frac{4c - p[\tau_H - \tau_L + 2c]}{a(\sigma_H) + a(\sigma)(1-p)}\right]\right]}.
\end{aligned}
$$

The low type mixes, and the high type rejects m. The leader accepts m if the enemy accepts, with beliefs $\lambda_1 = 1$.

The leader's beliefs upon observing the enemy reject are

$$\lambda_2 = \frac{2c - s_G^* a(\sigma_H)}{\tau_H - \tau_L}$$
$$= \frac{2c - a(\sigma_H)\left[\frac{4c - p[\tau_H - \tau_L + 2c]}{a(\sigma_H) + a(\sigma)(1-p)}\right]}{\tau_H - \tau_L}.$$

By Lemma 3, the leader must mix with the following probability r for the low type will to be indifferent:

$$r = \frac{2c - a(\sigma)\left[\frac{4c - p[\tau_H - \tau_L + 2c]}{a(\sigma_H) + a(\sigma)(1-p)}\right]}{\tau_H - \tau_L}.$$

The leader raises with concessions:

$$\delta = \sigma_H - m$$
$$= \tau_H - \tau_L - 2c + a(\sigma)\left[\frac{4c - p[\tau_H - \tau_L + 2c]}{a(\sigma_H) + a(\sigma)(1-p)}\right].$$

The audience sanctions with probability s_G^* and beliefs $\alpha_H^m = 0$, $\alpha_L^m = \frac{1}{2}$, $\alpha_L^r + \alpha_H^r = \frac{1}{2}$. The probability of war is

$$P(war) = \frac{(1-p)(\tau_H - m - c)}{\tau_H - \tau_L - 2c + s_G^* a(\sigma_H)}$$
$$= \frac{(1-p)\left(\tau_H - \tau_L - 2c + a(\sigma)\left[\frac{4c - p[\tau_H - \tau_L + 2c]}{a(\sigma_H) + a(\sigma)(1-p)}\right]\right)}{\tau_H - \tau_L - 2c + a(\sigma_H)\left[\frac{4c - p[\tau_H - \tau_L + 2c]}{a(\sigma_H) + a(\sigma)(1-p)}\right]}$$
$$= 1 - p.$$

□

Proposition 3 (Region III: Sanctions)**.** *When $p \in (p_H^*, p_L^*)$, the perfect Bayesian equilibrium for any pair $(p, a(\sigma_H))$ is as follows:*

1. *If the audience is weak, $a(\sigma_H) < 2c$, the mediator offers $m^* = \overline{\sigma_L} = \tau_L + c - a(\sigma)$, and the leader is sanctioned, $s^* = 1$. The probability of war is $\frac{(1-p)[\tau_H - \tau_L - 2c + a(\sigma)]}{\tau_H - \tau_L - 2c + a(\sigma_H)} < 1 - p$.*

2. *If the audience is strong, $a(\sigma_H) > 2c$, a high type is likely, $p < \frac{1}{2}$, and the sanction for accepting the mediator's offer is not too high, $a(\sigma) \leq 2c - \frac{p(\tau_H - \tau_L)}{1-p}$, then the mediator offers $m^* = \widehat{\sigma_L} = \sigma_L + \frac{p(\tau_H - \tau_L)}{1-p}$. The low type accepts, $q_L^* = 1$, the high type rejects. If the enemy rejects, the leader raises with probability $r^* = \frac{p}{1-p}$ and concessions $\delta^* = \frac{(1-2p)(\tau_H - \tau_L)}{1-p}$ with beliefs $\lambda_2 = 0$. The audience sanctions with probability $s^* = \frac{2c}{a(\sigma_H)}$ with beliefs $\alpha_L^m = \alpha_H^r = \frac{1}{2}$, $\alpha_L^r = \alpha_H^m = 0$. The probability of war is $1 - 2p$.*

3. *Otherwise, the mediator offers $m^* = \overline{\sigma_L} = \tau_L + c - s^* a(\sigma)$. The audience sanctions with probability $s^* = \frac{4c - p[\tau_H - \tau_L + 2c]}{a(\sigma_H) + a(\sigma)(1-p)}$ and beliefs $\alpha_L^m = \alpha_L^r + \alpha_H^r = \frac{1}{2}$, and $\alpha_H^m = 0$. The probability of war is $1 - p$.*

In each case, the leader accepts the mediator's offer with beliefs $\lambda_1 = 1$, and both types accept a raised offer.

In equilibria 1 and 3, the low type accepts with probability $q_L^* = \frac{p(\tau_H - \tau_L) - 2c + s^* a(\sigma_H)}{p[\tau_H - \tau_L - 2c + s^* a(\sigma_H)]}$, the high type rejects. If the enemy rejects, the leader believes it is a low type with probability $\lambda_2 = \frac{2c - s^* a(\sigma_H)}{\tau_H - \tau_L}$, and raises with probability $r^* = \frac{2c - s^* a(\sigma)}{\tau_H - \tau_L}$ and concessions $\delta^* = \tau_H - \tau_L - 2c + s^* a(\sigma)$.

Proof of Proposition 3. To summarize Lemmas 8, 9, and 10, there are three options in the Region III.

- Lemma 8: The mediator offers $\overline{\sigma_L}$, which is the most the leader will accept and the leader is sanctioned, $s = 1$. This is possible only if the audience is weak, $a(\sigma_H) < 2c$. The probability of war is less than $1 - p$.

- Lemma 9: The mediator offers $\widehat{\sigma_L}$ that corresponds to the maximum probability the indifferent audience will sanction, $s_H^* = \frac{2c}{a(\sigma_H)}$. Here the price of the mediator's offer is strictly less than the maximum settlement the leader will accept against the low type, $\widehat{\sigma_L} < \overline{\sigma_L}$. This is possible only if a high type is likely, $p < \min\{p_L^*, \frac{1}{2}\}$, the audience is sufficiently strong, $a(\sigma_H) > 2c$, and sanctioning for accepting the mediator's offer is not too high, $a(\sigma) \leq 2c - \frac{p(\tau_H - \tau_L)}{1-p}$. The probability of war is $1 - 2p$.

- Lemma 10: The mediator offers $\widehat{\sigma_L} = \overline{\sigma_L}$, which is the most the leader will accept that keeps the audience indifferent between sanctioning and not. The audience sanctions with s_G^*. The probability of war is $1 - p$.

Since the first and second options result in a strictly lower probability of war, the mediator prefers to make those offers when possible. These options are not simultaneously available, since the first option relies on a weak audience, $a(\sigma_H) < 2c$, and the second option relies on a strong audience, $a(\sigma_H) > 2c$. If these options are not available, then the mediator resorts to the third option.

Since none of these equilibria overlap, there is only one equilibrium for every pair $(p, a(\sigma_H))$: the equilibrium is unique.

What is necessary to maintain this equilibrium? To maintain this equilibrium, the mediator must not deviate to another offer m'. To see that the mediator will not deviate to a lower offer $m' < m^*$, recall that the probability of war is decreasing in m, and thus, the mediator will not deviate to a lower offer.

To prevent the mediator from deviating to a higher offer $m' > m^*$, note that if the mediator deviates to a higher offer, then in the first case the leader will reject this offer thereby increasing the probability of war. In the second case, the audience will no longer be indifferent, and thus since the leader is sanctioned, she will exit to war rather than raise. This also increases the probability of war. In the third case, the leader will reject the mediator's offer, again increasing the probability of war. Since in all three cases, a higher proposal increases the probability of war, the mediator will not deviate to a higher offer.

□

Proposition 4 (Negotiation)**.** *When $p < p_N^*$, the leader offers σ_H and both types accept, where $p_N^* = \frac{2c}{\tau_H - \tau_L + 2c}$. Otherwise, the leader offers σ_L, which risks war against the high type. The probability of war is $1 - p$.*

Proof. Since the leader knows she will face audience costs, the leader never makes an offer knowing that she will raise and pay audience costs. Thus, she makes the high offer if paying that high settlement is better than her odds of a low type accepting the low offer and war against a high type:

$$\begin{aligned}
U(\sigma_H) \geq & U(\sigma_L) \\
-\tau_H + c \geq & p(-\tau_L + c) + (1-p)(-\tau_H - c) \\
-\tau_H + c \geq & p(\tau_H - \tau_L + 2c) - \tau_H - c \\
p \leq & \frac{2c}{\tau_H - \tau_L + 2c} \equiv p_N^*.
\end{aligned}$$

When $p < p_N^*$, the leader makes a high offer that secures peace, and otherwise, she makes a low offer that risks war against the less likely high type. War occurs against the high type when the low offer is made, with probability $1 - p$. $\square$